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## CHARACTERIZING LOW LIGHT LEVEL SIGNALS WITH THE TRIGGERED DELAYED PHOTOCOUNT DISTRIBUTION

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### ABSTRACT

The theoretical expression of the first-order factorial moment of a triggered delayed photocount distribution and a description of its SNR have been developed. We explain the adventages of this thecnique, in comparison with the second-order correlation function, when we measured a temporal signal under low light level. We derived the theoretical errors involved in determining a parameter by measuring both functions for a signal with a periodically modulated intensity, since this kind of signals usually appear in spectroscopy. We conclude that the first-order factorial moment of a triggered delayed photocount distribution is a useful tool for characterizing a signal at low light level conditions.

## INTRODUCTION

The measurement of the first-order factorial moment (FOFM) of a triggered photocount distribution (TPCD)<sup>1</sup> is used for the analysis of a light beam because it can be easily measured and it contains the same information about the signal to be analyzed as the autocorrelation function  $g^{(2)}(\tau)$ . In a standard TPCD, where the measurement time ( $T$ ) starts immediately one pulse is registered, the FOFM only provide the information contained in  $g^{(2)}(0)$ <sup>2</sup>. But if we want to obtain the information about  $g^{(2)}(\tau)$  for every value of  $\tau$ , we have to evaluate the FOFM of a triggered delayed photocount distribution (TDPCD).

A TDPCD is a TPCD in which a time or space interval ( $\theta$ ) is introduced between the occurrence of one photoevent and the beginning of the counting time<sup>3</sup>. The triggering pulse which starts the counting process can be obtained from the signal to be analyzed (Auto-trigger) or from a different signal (Cross-trigger). The aim of this work is to characterize the use of the FOFM of a TDPCD as a method for signal characterization at low light level.

## THEORY

In order to compare the results obtained by using the FOFM of the TDPCD with the autocorrelation function, consider a temporal signal  $I(t)$

The photocount distribution  $P(n,\theta,T)$  of registered  $n$  photoelectrons during the time interval  $(T)$ , when the counting time is delayed by a time  $\theta$  from the occurrence of one process point, can be expressed as follows<sup>3</sup>:

$$P(n,\theta,T) = \langle I(t) [w(t+\theta,T)]^n \exp(-w(t+\theta,T)) \rangle / (\langle I \rangle n!) \quad [1]$$

where the mean value corresponds to a random average of  $w(t+\theta,t)$ , and

$$w(t+\theta,T) = \int_{t+\theta}^{t+\theta+T} I(t') dt' \quad [2]$$

It can be shown<sup>5</sup> that the expression of the experimental FOFM of a TDPCCD is:

$$N^{(1)}(\theta, T) = \langle I(t) w(t+\theta,T) \rangle / \langle I \rangle \quad [3]$$

The normalized FOFM will be:

$$n^{(1)}(\theta, T) = N^{(1)}(\theta, T) / \langle w \rangle \quad [4]$$

If we use a TPCD the value of  $\theta$  in Eq.(3) becomes zero.

The expression of the experimental autocorrelation function is:

$$g^{(2)}(\tau, T) = \langle w(t, T) w(t+\tau, T) \rangle / \langle w \rangle^2 \quad [5]$$

which is very similar to Eq.(4) for small counting times, where  $w(t,T) \approx I(t)T$ . Therefore, Eq. (4) are related with the expression of the theoretical autocorrelation function by a temporal derivate<sup>2</sup>.

We have developed an expresion for the variance of  $n^{(1)}(\theta, T)$ , by the same way that Saleh<sup>4</sup> used for evaluating the variance of the FOFM of a standard PCD:

$$\begin{aligned} \text{Var}[n^{(1)}(\theta, T)] &= (1/n)^2 \text{Var}(N^{(1)}) + (N^{(1)}/\bar{n}^2)^2 \text{Var}\bar{n} + \\ &+ 2(-N^{(1)}/\bar{n}^3)[(N^{(1)}\bar{n}) - \bar{n}\langle N^{(1)} \rangle] \end{aligned} \quad [6]$$

For small values of  $n$  ( mean number of photoevents per counting time) we obtain an approximate equation:

$$\text{Var}(n^{(1)}) \approx n^{(1)} / (N\bar{n}) \quad [7]$$

where  $N$  is the number of experiments used to evaluate it. Under the same conditions the following expresion of the variance of  $g^{(2)}(\tau, T)$  can be obtained<sup>4</sup>:

$$\text{Var}(g^{(2)}) \approx g^{(2)} / (N\bar{n}^2) \quad [8]$$

The SNR derived from Eqs.(7) and (8) will be:

$$\text{SNR}_n = (n^{(1)} N \bar{n})^{1/2} \quad [9]$$

$$\text{SNR}_g = (g N \bar{n}^2)^{1/2} \quad [10]$$

For small values of  $\bar{n}$   $\text{SNR}_h$  will be greater than  $\text{SNR}_g$ . So we can obtain a statistical function which contains the same information about the signal as the autocorrelation function but has the advantages of a easier processing procedure and better SNR.

We suggest a new experimental way for measuring the FOFM of a TDPSCD. It is to use a multichannel analyzer, auto or cross-triggered, and so, for each trigger pulse, we obtain a set of values of  $n^{(1)}(\theta, T)$  for different values of  $\theta$ .

If we want to know the error in determining a parameter  $\alpha$  from the measurement of a function  $f(\alpha, t)$  for various values of the delay interval  $t$ , we have to use the expression:

$$\text{Var } \alpha = \left( \sum_{i=1}^L \left[ (df(\alpha, t_i)/d\alpha)^2 / \text{Var } f(\alpha, t_i) \right] \right)^{-1} \quad [11]$$

## APPLICATION

We shall now evaluate the relative errors obtained, when the measured function is  $n^{(1)}(\theta, T)$  and  $g^{(2)}(t, T)$  respectively, in determining the parameter  $\alpha = M$  for the light signal whose intensity is periodically modulated:

$$I(t) = \bar{I} (1 + M \cos \omega t) \quad [12]$$

where  $\omega = 2\pi/P$ ,  $P$  is the period of the signal,  $M$  is the modulation factor and  $\bar{I}$  the mean value of the intensity.

Taking into account the fact that

$$w(t, T) = \bar{I} T [1 + \cos((t+T/2)\omega)] \quad [13]$$

and using Eqs. (4) and (5) we obtain :

$$g^{(2)}(t, T) = 1 + (A^2/2) \cos \omega t \quad [14]$$

$$n^{(1)}(\theta, T) = 1 + (M A/2) \cos \omega(\theta+T/2) \quad [15]$$

where

$$A = M \sin(\omega T/2) / (\omega T/2) \quad [16]$$

By using Eqs. (7) and (8), and by evaluating the derivatives which appear in Eq. (11), we have represented the relative error in percent involved in determining  $M$  by measuring  $n^{(1)}(\theta, T)$  as function of:

a) the number of channels  $L = P/t$  ranging from 5 to 35 in steps of 5, and the value of  $M$  varying from 0.1 to 0.7 with steps of 0.1. The value of the mean number of photocounts per counting time is fixed :  $\bar{n} = 10^{-3}$  counts (Fig (1)).

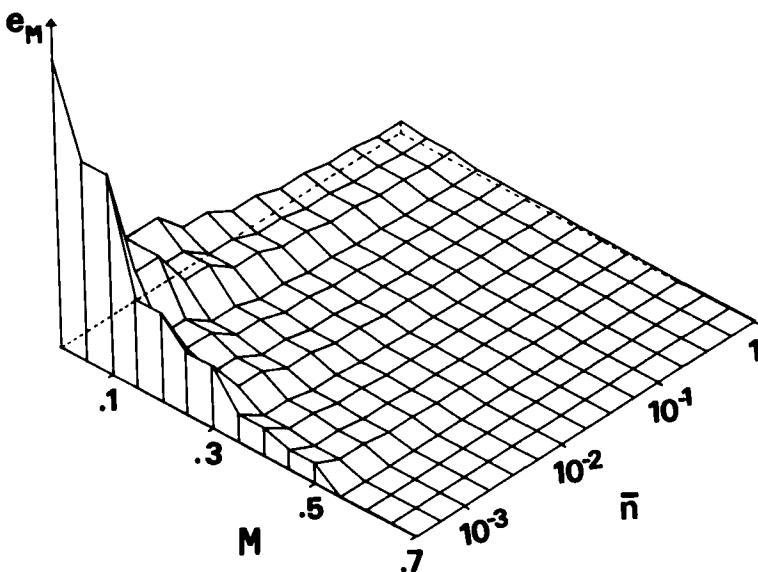


Fig. 1.- Relative error ( $e_M$ ) in percent involved in determining the parameter  $M$  of the signal  $I(t) = \bar{I} (1+M \cos(\omega t))$  by measuring  $n^{(1)}(\theta, T)$  versus the number of channels ( $L$ ), and the modulation factor ( $M$ ) for a value of  $\bar{n}=10^{-3}$  photocounts per counting time.

b) the number of photocounts per counting time  $\bar{n}$  ( $\bar{n}$  from  $10^{-3}$  to 1) and  $M$  ( $M$  in the range from 0.1 to 0.7 in steps of 0.1), fixed  $L=50$  channels (Fig (2)).

It can be seen that the behaviour of the error involved in obtaining  $M$  by measuring  $n^{(1)}(\theta, T)$  and  $g^{(2)}(t, T)$  is the same, but the values for  $g^{(2)}(t, T)$  is greater than those obtained for  $n^{(1)}(\theta, T)$  10 times in the case (a) and up to 30 times in the case (b) for small values of  $\bar{n}$ .

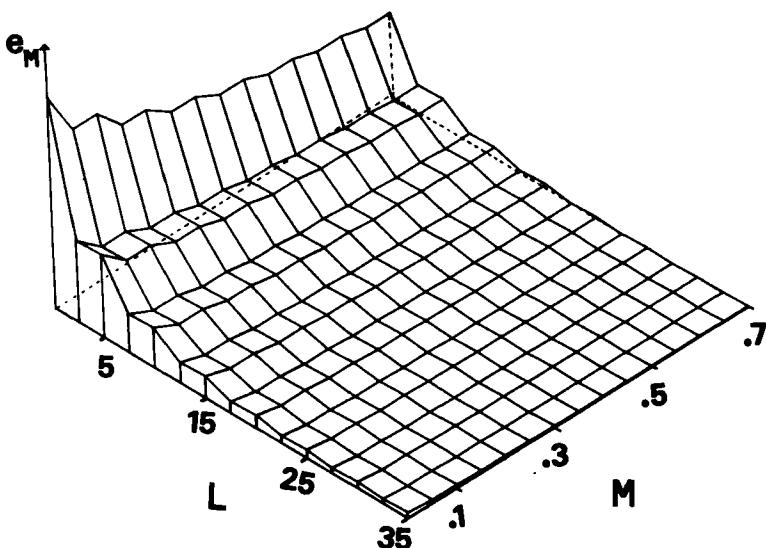


Fig. 2.- Relative error ( $e_M$ ) in percent involved in determining the parameter  $M$  of the signal  $I(t) = \bar{I} (1+M \cos(\omega t))$  by measuring  $n^{(1)}(\theta, T)$  versus the number of photocounts per counting time ( $\bar{n}$ ), and the modulation factor ( $M$ ) for a value of  $L=50$  channels.

### CONCLUSIONS

In analyzing a deterministic light signal at low light level, the normalized FOFM of a TDPSCD contains the same information as the normalized second-order autocorrelation function, but its measurement produce smaller errors in the determination of the characterizing parameters of the light beam. We can conclude that the measurement of  $n^{(1)}(\theta, T)$  is an useful method for characterizing low light level signals, since, in this case, it presents better results than  $g^{(2)}(t, T)$ .

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