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CHARACTERIZING LOW LIGHT LEVEL SIGNALS WITH THE TRIGGERED DELAYED PHOTOCOUNT DISTRIBUTION

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ABSTRACT

The theoretical expression of the first-order factorial moment of a triggered delayed photocount distribution and a description of its SNR have been developed. We explain the advantages of this technique, in comparison with the second-order correlation function, when we measured a temporal signal under low light level. We derived the theoretical errors involved in determining a parameter by measuring both functions for a signal with a periodically modulated intensity, since this kind of signals usually appear in spectroscopy. We conclude that the first-order factorial moment of a triggered delayed photocount distribution is a useful tool for characterizing a signal at low light level conditions.

INTRODUCTION

The measurement of the first-order factorial moment (FOFM) of a triggered photocount distribution (TPCD)¹ is used for the analysis of a light beam because it can be easily measured and it contains the same information about the signal to be analyzed as the autocorrelation function $g^{(2)}(\tau)$. In a standard TPCD, where the measurement time (T) starts immediately one pulse is registered, the FOFM only provide the information contained in $g^{(2)}(0)^2$. But if we want to obtained the information about $g^{(2)}(\tau)$ for every value of τ , we have to evaluate the FOFM of a triggered delayed photocount distribution (TDPCD).

A TDPCD is a TPCD in which a time or space interval (θ) is introduced between the occurrence of one photoevent and the beginning of the counting time³. The triggering pulse which starts the counting process can be obtained from the signal to be analyzed (Auto-trigger) or from a different signal (Cross-trigger). The aim of this work is to characterize the use of the FOFM of a TDPCD as a method for signal characterization at low light level.

THEORY

In order to compare the results obtained by using the FOFM of the TDPCD with the autocorrelation function, consider a temporal signal $I(t)$

The photocount distribution $P(n, \theta, T)$ of registered n photoelectrons during the time interval (T) , when the counting time is delayed by a time θ from the occurrence of one process point, can be expressed as follows³:

$$P(n, \theta, T) = \langle I(t) [w(t+\theta, T)]^n \exp(-w(t+\theta, T)) \rangle / (\langle I \rangle n!) \quad [1]$$

where the mean value corresponds to a random average of $w(t+\theta, t)$, and

$$w(t+\theta, T) = \int_{t+\theta}^{t+\theta+T} I(t') dt' \quad [2]$$

It can be shown⁵ that the expression of the experimental FOFM of a TPCD is

$$N^{(1)}(\theta, T) = \langle I(t) w(t+\theta, T) \rangle / \langle I \rangle \quad [3]$$

The normalized FOFM will be :

$$n^{(1)}(\theta, T) = N^{(1)}(\theta, T) / \langle w \rangle \quad [4]$$

If we use a TPCD the value of θ in Eq.(3) becomes zero.

The expression of the experimental autocorrelation function is :

$$g^{(2)}(\tau, T) = \langle w(t, T) w(t+\tau, T) \rangle / \langle w \rangle^2 \quad [5]$$

which is very similar to Eq.(4) for small counting times, where $w(t,T) \cong I(t) T$. Therefore, Eq. (4) are related with the expression of the theoretical autocorrelation function by a temporal derivate².

We have developed an expression for the variance of $n^{(1)}(\theta, T)$, by the same way that Saleh⁴ used for evaluating the variance of the FOFM of a standard PCD :

$$\begin{aligned} \text{var}[n^{(1)}(\theta, T)] = & (1/n)^2 \text{Var}(N^{(1)}) + (N^{(1)}/\bar{n}^2)^2 \text{Var} \bar{n} + \\ & + 2(-N^{(1)}/\bar{n}^3) [\langle N^{(1)} \bar{n} \rangle - \bar{n} \langle N^{(1)} \rangle] \end{aligned} \quad [6]$$

For small values of n (mean number of photoevents per counting time) we obtain an approximate equation :

$$\text{var}(n^{(1)}) \cong n^{(1)} / (N \bar{n}) \quad [7]$$

where N is the number of experiments used to evaluate it. Under the same conditions the following expresion of the variance of $g^{(2)}(\tau, T)$ can be obtained⁴ :

$$\text{Var}(g^{(2)}) \cong g^{(2)} / (N \bar{n}^2) \quad [8]$$

The SNR derived from Eqs.(7) and (8) will be :

$$\text{SNR}_n = (n^{(1)} N \bar{n})^{1/2} \quad [9]$$

$$\text{SNR}_g = (q N \bar{n}^2)^{1/2} \quad [10]$$

For small values of \bar{n} SNR_n will be greater than SNR_g . So we can obtain a statistical function which contains the same information about the signal as the autocorrelation function but has the advantages of a easier processing procedure and better SNR.

We suggest a new experimental way for measuring the FOFM of a TDPCD. It is to use a multichannel annalyzer, auto or cross-triggered, and so, for each trigger pulse, we obtain a set of values of $n^{(1)}(\theta, T)$ for different values of θ .

If we want to know the error in determining a parameter α from the mesurament of a function $f(\alpha, \tau)$ for various values of the delay interval τ , we have to use the expresion :

$$\text{Var } \alpha = \left(\sum_{i=1}^L \left[(df(\alpha, \tau_i) / d\alpha)^2 / \text{Var } f(\alpha, \tau_i) \right] \right)^{-1} \quad [11]$$

APLICATION

We shall now evaluate the relative errors obtained, when the measured function is $n^{(1)}(\theta, T)$ and $g^{(2)}(\tau, T)$ respectively, in determining the parameter $\alpha=M$ for the light signal whose intensity is periodically modulated :

$$I(t) = \bar{I} (1 + M \cos \omega t) \quad [12]$$

where $\omega = 2\pi/P$, P is the period of the signal, M is the modulation factor and \bar{I} the mean value of the intensity.

Taking into account the fact that

$$w(t, T) = \bar{I} T [1 + \cos((t+T/2)\omega)] \quad [13]$$

and using Eqs. (4) and (5) we obtain :

$$g^{(2)}(\tau, T) = 1 + (A^2/2) \cos \omega \tau \quad [14]$$

$$n^{(1)}(\theta, T) = 1 + (MA/2) \cos \omega(\theta+T/2) \quad [15]$$

where

$$A = M \sin(\omega T/2) / (\omega T/2) \quad [16]$$

By using Eqs. (7) and (8), and by evaluating the derivatives which appear in Eq. (11), we have represented the relative error in percent involved in determining M by measuring $n^{(1)}(\theta, T)$ as function of:

a) the number of channels $L = P/\tau$ ranging from 5 to 35 in steps of 5, and the value of M varying from 0.1 to 0.7 with steps of 0.1. The value of the mean number of photocounts per counting time is fixed : $\bar{n} = 10^{-3}$ counts (Fig (1)).

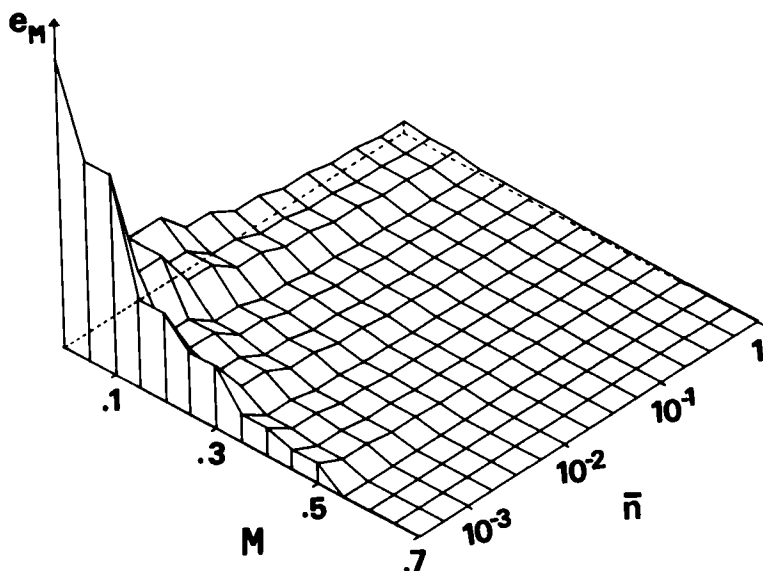


Fig. 1.- Relative error (e_M) in percent involved in determining the parameter M of the signal $I(t) = \bar{I} (1 + M \cos(\omega t))$ by measuring $n^{[1]}(\theta, T)$ versus the number of channels (L), and the modulation factor (M) for a value of $\bar{n} = 10^{-3}$ photocounts per counting time.

b) the number of photocounts per counting time \bar{n} (\bar{n} from 10^{-3} to 1) and M (M in the range from 0.1 to 0.7 in steps of 0.1), fixed $L=50$ channels (Fig (2)).

It can be seen that the behaviour of the error involved in obtaining M by measuring $n^{[1]}(\theta, T)$ and $g^{(2)}(\tau, T)$ is the same, but the values for $g^{(2)}(\tau, T)$ is greater than those obtained for $n^{[1]}(\theta, T)$: 10 times in the case (a) and up to 30 times in the case (b) for small values of \bar{n} .

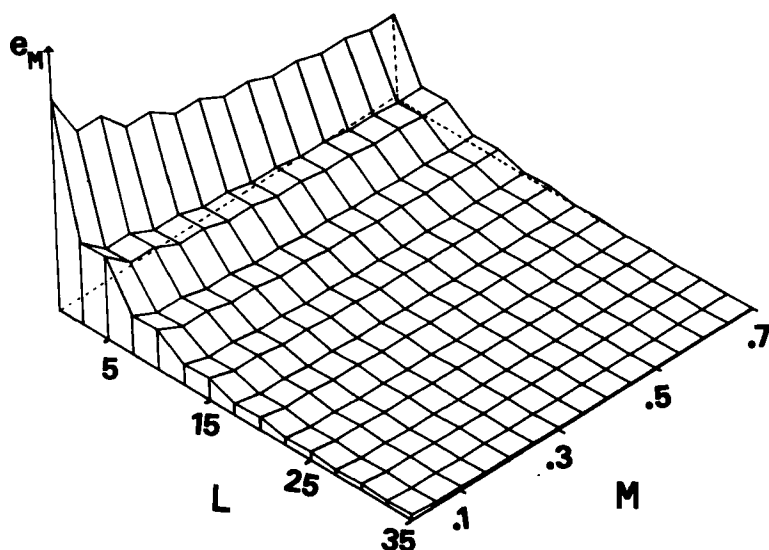


Fig. 2.- Relative error (e_M) in percent involved in determining the parameter M of the signal $I(t) = \bar{I}(1 + M \cos(\omega t))$ by measuring $n^{(1)}(\theta, T)$ versus the number of photocounts per counting time (\bar{n}), and the modulation factor (M) for a value of $L=50$ channels.

CONCLUSIONS

In analyzing a deterministic light signal at low light level, the normalized FOFM of a TDPCD contains the same information as the normalized second-order autocorrelation function, but its measurement produce smaller errors in the determination of the characterizing parametrers of the light beam. We can conclude that the measurement of $n^{(1)}(\theta, T)$ is an useful method for characterizing low light level signals, since, in this case, it presents betters results than $g^{(2)}(\tau, T)$

REFERENCES

1- L. Basano, P. Ottonello : "Triggering techniques in triggered photon-counting " , Appl. Opt. , 21, 3677 (1982) .

2- T. Yoshimura , T. Nakajima , N. Wakabayashi : "Statistical properties of triggered photocounting distribution " , Appl. Opt. , 20, 2993 (1981) .

3- C. Bendjaballah , F. Perot : "Statistical properties of intensity-modulated coherent radiation . Theoretical and experimental aspects " , J. Appl. Phys. , 44,11 (1973) .

4- B. A.Saleh : "Photoelectron Statistics " , (Springer- Verlag , Berlin , (1978) .

5-M. A. Rebolledo ,J.M. Alvarez, J.C. Amaré, "Analysis of a light beam with a periodically modulated intensity by triggered photon counting". J. Opt. Soc. Am. ,3,1,(1986).

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